Research Statement

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I have a broad interest in arithmetic geometry and related subjects and I especially enjoy developing and applying tools from varied fields to questions in arithmetic geometry (and vice-versa). Most notably, Andrew O'Desky and I (Project 1.1) develop a combinatorial framework to compute motivic invariants of certain natural moduli space in algebraic geometry. In the other direction, my ongoing work (Project 1.2) with Colin Crowley, Connor Simpson and Botong Wang) tackles an outstanding conjecture on matroids, taking inspiration from algebro-geometric methods.

Within algebraic geometry, a major theme of my work concerns moduli spaces. I have also recently developed an interest in arithmetic dynamics and I combine these two interests to study the dynamics of Hecke correspondences on moduli spaces and more specifically, "unlikely and just likely intersections" (Section 2).

I give a brief synopsis of my various projects here and expand on them in the rest of this Statement.

- **Project 1.1**: [OG22] is joint work with Andrew O'Desky. It is concerned with the development of novel combinatorial tools in order to compute motivic invariants of moduli spaces of polynomials with a fixed factorization type.
- **Project 1.2**: This is a project in progress (joint with Colin Crowley, Connor Simpson and Botong Wang). We aim to prove an equivariant version of various log concavity statements related to matroids.
- **Project 2**: This project concerns the question of "unlikely and just likely intersections" on Shimura varieties. In [G.22b], I prove the first general results towards a conjecture on unlikely intersections of arbitrary subvarieties on Shimura varieties formulated by Shou Wu-Zhang's AIM group in the context of a product of modular curves. In [**?**], Ananth Shankar, Qiao He and I prove the conjecture for Hilbert modular varieties over a finite field (with mild restrictions).

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- **Project 3.1**: [G.22a] proves a novel result in non-abelian Iwasawa theory in order to prove a geometric statement about the convergence of certain characteristic polynomials in a tower of curves. At its core is an elementary statement regarding a matrix generalization of Fermat's little theorem.
- **Project 3.2:** In joint work with Yifan Wei and John Yin, we generalize and prove a conjecture [BCFG22, Conjecture 1.2] about p-adic densities of polynomials with a given factorization type. In doing so, we prove a version of Chebotarev's theorem for non-archimedean local fields like $\mathbb{Q}_p$.

1 COMBINATORICS

1.1 *Configuration spaces, graded spaces, and polysymmetric functions*

Recall the following classical result of Gauss.

Theorem 1 (Gauss). *The number of monic, irreducible degree d polynomials in* 1 *variable over the finite field $\mathbb{F}_q$ is given by*

$$N_{d,1}(q) = \sum_{m|d} \mu(m) q^{d/m}.$$

One is naturally led to ask the same question when the polynomials are allowed to have many variables. In this context, one needs to make a distinction between *arithmetic* and *geometric* irreducibility[1]. For technical reasons, we find it more useful to work with homogeneous polynomials in $n+1$ variables and we define $N^{\text{arith}}_{d,n}(q)$ (respectively $N^{\text{geom}}_{d,n}(q)$) to be the number of arithmetic (respectively geometric) irreducible degree d polynomials in $n+1$ variables up to scaling by $\mathbb{F}_q^\times$. We prove

Theorem 2. *We have the following generalizations of Gauss' result.*

$$N^{\text{arith}}_{d,n}(q) = \frac{1}{d}\sum_{m|d} m\mu(\tfrac{d}{m}) \sum_{\lambda\vdash m} (-1)^{\ell-1}\frac{(\ell-1)!}{n_1!\cdots n_m!}\prod_{k=1}^{m}\left(1+q+\cdots+q^{\binom{n+k}{k}-1}\right)^{n_k},$$

$$N^{\text{geom}}_{d,n}(q) = \frac{1}{d}\sum_{m|d} m\mu(\tfrac{d}{m}) \sum_{\lambda\vdash m} (-1)^{\ell-1}\frac{(\ell-1)!}{n_1!\cdots n_m!}\prod_{k=1}^{m}\left(1+q^{d/m}+\cdots+q^{d/m[\binom{n+k}{k}-1]}\right)^{n_k}.$$

In proving this result, it is useful to consider the moduli space $M_{d,n}$ of irreducible homogeneous polynomials of degree d in $n+1$ variables (up to scaling). It is an open subset of the projective space $X_{d,n} = \mathbb{P}^{\binom{n+d}{d}-1}$ and crucially, polynomial multiplication induces maps $X_{d,n}\times X_{e,n}\to X_{d+e,n}$ that makes the collection $\{X_{d,n} : n \geq 1\}$ into

1 i.e., irreducibility over $\mathbb{F}_q$ versus irreducibility over $\overline{\mathbb{F}}_q$.

a graded monoid space. Whenever we have such a collection of spaces satisfying "unique factorization", [OG22, Theorem 0.1] compute the class of $M_{d,n}$ in terms of the class of $X_{d.n}$ in the Grothendieck ring of varieties which can be considered as a "universal" euler characteristic.

In order to prove these general results, we define $P\Lambda$, the ring of *polysymmetric functions* as an extension of the classical theory of symmetric functions. Very briefly, one can consider polsymmetric functions to the theory of symmetric functions with weighted variables [OG22, Part 1]. We extend several constructions from the classical theory including a generalization of partitions called types which record the possible factorizations of a degree d polynomial. Using the notion of types, we define analogues of the classical elementary, homogeneous, monomial and power symmetric bases. Moreover, we find a new symmetry and partial order not found in the classical theory.

The final ingredient that is needed is the notion of *graded plethysm*, extending the classical notion of plethysm on Lambda rings. Lambda rings were originally defined by Grothendieck in the context of the Grothendieck-Riemann-Roch theorem. The prototypical example of a Lambda ring is the ring of representations of a group and the lambda operations axiomatize the operations of symmetric and exterior products.

The Grothendieck ring of motives $K_0(\text{var})$ also has the structure of a Lambda ring but it is deficient in one important respect - it is not a *special* lambda ring. We use the ring of polysymmetric functions $P\Lambda$ as a replacement for $K_0(\text{var})$ in order to prove our theorems.

Moreover, we can use types to give a stratification of $X_{d,n}$ parametrizing the space of polynomials with a given factorization type. We give a recursive formula for these strata [OG22, Section 7] and show that they are closely related to the graded plethysm operations in a natural "combinatorial" setting [OG22, Section 8].

1.2 *Equivariant log concavity*

There has been much recent work in proving the log concavity of sequences, inspired by techniques from algebraic geometry ([AHK18],[HW17] among many others). For a survey of the field, see [Huh22] and [Kal22].

To take a concrete example, consider a finite collection of vectors $\mathcal{V} \subset k^n$ in a vector space over a field k. For each positive integer k, we define $\mathcal{I}_k \subset \mathcal{P}(\mathcal{V})$ to be the collection of subsets of $\mathcal{V}$ of size k consisting of linearly independent vectors. So for instance, $\mathcal{I}_1$ has elements corresponding to the non-zero elements in $\mathcal{V}$. If $\mathcal{V}$ is itself a collection of linearly independent vectors, then $\mathcal{I}_k$ is simply the collection of all subsets of $\mathcal{V}$ of size k. More generally, we can consider any *matroid* and consider the collection of independent sets of size k for each k as above.

Mason [Mas72] conjectured that the sequence $(\#\mathcal{I}_1, \#\mathcal{I}_2, \dots)$ is a log-concave sequence[2] for any matroid. This conjecture (and some stronger versions) have been recently proven by [AHK18],[ALGV18],[BH18],[BH20] among others. We consider a strengthening of this conjecture in a different direction. Returning to our example above, let G be a subgroup of the permutations of $\mathcal{V}$ that preserve the notion of linear independence (and for an arbitrary matroid, we define G to be the automorphisms of the ground set that preserve the notion of independence). Then G will act on the sets $\mathcal{I}_k$ and we define $P_k := k[\mathcal{I}_k]$ to be the corresponding permutation representations. One is naturally led to the following conjecture (Mason's conjecture corresponds to the case where G is the trivial group).

Conjecture 3. *For any $1 \leq a \leq b \leq c \leq d$ with $a + d = b + c$, there is a G equivariant injection $P_a \otimes P_d \to P_b \otimes P_c$.*

One can make analogous conjectures for various other algebraic invariants associated to a matroid such as the Orlik-Solomon algebra. [GPY17] and [MMPR21] formulate such conjectures, prove it in some very special cases and provide numerical evidence for the conjecture in general.

In our work so far, we have a refinement of the above family of conjectures where we define an *explicit* G-equivariant map $P_a \otimes P_d \to P_b \otimes P_c$ and it remains to prove that this is an injective map. If successful, our approach would give an alternate proof of Mason's conjecture bypassing the earlier work on this question.

2 UNLIKELY INTERSECTIONS OF SUBVARIETIES ON A SHIMURA VARIETY

There has been much recent interest on questions of *just likely and unlikely intersections* on Shimura varieties. As the simplest case, consider two families of elliptic curves $\mathcal{E}, \mathcal{F}$ relative to C, a curve over a finite field or the ring of integers of a number field (possibly localized at some finite set of primes). In these cases, it is known under mild restrictions on the two families that there are infinitely many geometric points $x \in C$ such that the fibers $\mathcal{E}_x$ and $\mathcal{F}_x$ are isogenous to each other ([CO06, Proposition 7.3],[Cha14]).

To better understand this result, let us reformulate it in terms of moduli spaces. A family of elliptic curves over C gives rise to a map

$$\begin{aligned} C &\to X(1) \\ x &\to j(\mathcal{E}_x) \end{aligned}$$

where $X(1)$ is the modular curve so that our set up above corresponds to a map $f : C \to X(1) \times X(1)$. The geometric points $x \in C$ such that $\mathcal{E}_x$ is isogenous to $\mathcal{F}_x$ correspond exactly to the intersection of $f(C)$ with $T_N(\Delta)$, a Hecke translate of the

2 i.e., $\#\mathcal{I}_k^2 \geq \#\mathcal{I}_{k-1}\#\mathcal{I}_{k+1}$.

diagonal $\Delta \subset X(1)^2$. Since $f(C)$ and $T_N(\Delta)$ have complimentary dimension, we expect there to be finitely many points in this intersection for any fixed N and the question is about the behaviour as $N \to \infty$. Such results have been generalized to families of abelian surfaces ([MST22b],[ST20]) and K3 surfaces ([MST22a],[SSTT19]).

Briefly, these papers consider the intersection of a curve C with a "special divisor" D in a Shimura variety and show that the number of distinct points in the intersection $C \cap T_N(D)$ grows without bound for $\{T_N : N \geq 1\}$ a sequence of Hecke translations. To do this, they bound the local intersection number $(C \cap T_N(D))_x$ uniformly in N but on the other hand, also show that the global intersection number $C \cap T_N(D) \to \infty$ as $N \to \infty$. These two facts together prove the desired result. Crucially, they provide the local bound by appealing to the moduli interpretation of the associated Shimura variety and applying deformation theoretic techniques while the global intersection number computation is related to the Fourier coefficients of a modular form using the "specialness" of D.

Despite the proofs relying heavily on such moduli theoretic considerations, these problems can be naturally phrased for arbitrary subvarieties of a Shimura variety. Indeed, this was done by an AIM group working under Shou-Wu Zhang (Section 3.4).

Conjecture 4. *Let $\mathcal{S}$ denote a simple Shimura variety (over some base), and suppose that V, W are generically ordinary subvarieties having complementary dimension. Then the set of points in V isogenous to some point of W is Zariski-dense in C. Further, the subset of $V \times W$ consisting of pairs of isogenous points is dense in $V \times W$.*

In this generality, the conjecture is known in very few cases. Perhaps most significantly, it is known in complete generality in the geometric case, i.e., over $\mathbb{C}$ by [TT21]. The aforementioned work [Cha14] deals with the case where $\mathcal{S} = Y(1)$ relative to $\mathcal{O}_k$, the ring of integers of a number field (using the same strategy of proof as described before). In all other cases, it seems very hard to directly extend the older methods of proof to tackle the above conjecture when V, W are allowed to be arbitrary subvarieties.

We provide the first evidence for the above conjecture for arbitrary subvarieties in an *arithmetic* setting.

2.1 *Families of totally split abelian varieties*

In [G.22b, Theorem 6], I prove the above conjecture (suitably modified) for $\mathcal{S} = Y(1)^n$. This result naturally leads to the question where $\dim V + \dim W \ll \dim \mathcal{S}$. In the number field setting, the above is impossible if one of V, W is special by the Zilber-Pink conjecture (see [HP16, Conjecture 2.2]).

Over a finite field, [ST18] formulates the following heuristic in this direction. Let $\mathrm{Is}_n(m)$ be the set of isogeny classes corresponding to the abelian varieties parameterized by $Y(1)^n(\mathbb{F}_{q^m})$. [DH98, Theorem 1.1] shows that the size of the set $\mathrm{Is}_n(m)$ is about $q^{nm/2}$ (as $q^n \to \infty$) . It seems reasonable to treat the map $i : Y(1)^n(\mathbb{F}_{q^m}) \to \mathrm{Is}_n(m)$

sending a point $x \in Y(1)^n(\mathbb{F}_q)$ to the isogeny class of the corresponding abelian variety A_x as a *random* map and suppose that every isogeny class is of size $q^{nm/2}$ (ignoring problems related to supersingularity and the like). Let $V, W \subset Y(1)^n$ now be curves. For $n \geq 2$ and m large enough, one would expect $|i(V(\mathbb{F}_{q^m}))| \sim q^m$ and $|i(V(\mathbb{F}_{q^m})) \cap i(W(\mathbb{F}_{q^m}))| \sim q^{2m}/q^{nm/2}$. Summing over all m, one expects that the number of points on V geometrically isogenous to a point on W is approximately $(1 - q^2/q^{n/2})^{-1}$ which is a finite quantity (for $n \geq 5$). Surprisingly, I find two infinite families of counter examples to this heuristic in [G.22b, Theorems 13,14] - there exist curves C_n, D_n in $X(1)^n$ (for n arbitrarily large) such that there are infinitely many points on C_n isogenous to some point on D_n.

2.2 *Abelian surfaces with real multiplication*

In [**?**, Theorem 1.1], Ananth, Qiao and I prove the conjecture for $\mathcal{S}$ a Hilbert modular surface over $\mathbb{F}_p$ with the restriction that p splits in the ring of endomorphisms. In this case, $\mathcal{S}$ does not have a global product structure but we use our assumption on p, endomorphism ring to construct a *local* splitting of the Frobenius in order to complete the argument.

As a by-product of our analysis, we also compute the change of Faltings height under isogeny for Abelian surfaces parametrized by such $\mathcal{S}$.

3 p-ADIC ANALYSIS

3.1 *On the variation of the Frobenius in a non abelian Iwasawa tower*

The eigenvalues of the Frobenius on the étale cohomology of a smooth, projective variety over a finite field carry significant arithmetic information. By the Weil conjectures, these eigenvalues are algebraic integers and their absolute values under any complex embedding are understood.

In [G.22a], I draw inspiration from Iwasawa theory to study the asymptotic behaviour of these eigenvalues in an "Iwasawa tower" and in particular, show that there is a strong ℓ-adic convergence statement to be made in many natural examples. Consider the following example.

Example 1. Consider the tower defined by the smooth projective models corresponding to the equations

$$C_n : Y^2 = X^{3^n} + 1 \text{ over } \mathbb{F}_2$$

with maps $C_{n+1} \to C_n$ defined by $(X, Y) \to (X^3, Y)$. The characteristic polynomial of σ_2 on $H^1_{\text{ét}}(\overline{C}_n, \mathbb{Z}_\ell)$ is

$$f_n(x) := \det\left(1 - \sigma_2 x | H^1_{\text{ét}}(\overline{C}_n, \mathbb{Z}_\ell)\right) = \prod_{i=1}^{n}(1 + x^{2\cdot 3^{i-1}} 2^{3^{i-1}}).$$

Note that $f_{n-1}(x)$ divides $f_n(x)$ and the inverse roots of $g_n(x) = f_n(x)/f_{n-1}(x)$ are of the form $-\sqrt{2}\zeta$ for ζ a root of unity of order $2 \cdot 3^{n-1}$. I show that for n sufficiently large, the normalized (so that the complex absolute values is 1) roots of $g_{n+1}(x)$ are exactly all possible ℓ roots of the normalized roots of $g_n(x)$.

In fact, I prove the same statement for towers of Fermat curves (from which the above follows) and Artin-Schreier curves.

This prompts the question of what happens in a more general context. For instance, one could take a map $f : C \to \mathbb{P}^1$ or $f : C \to A$ for A an abelian variety of dimension d and pull back by the following diagrams:

$$\begin{array}{ccc} C_n & \xrightarrow{f_n} & \mathbb{P}^1 \\ \downarrow{\scriptstyle \pi_n} & & \downarrow{\scriptstyle t \to t^{\ell^n}} \\ C & \xrightarrow{f} & \mathbb{P}^1 \end{array} \qquad \mathit{or} \qquad \begin{array}{ccc} C_n & \xrightarrow{f_n} & A \\ \downarrow{\scriptstyle \pi_n} & & \downarrow{\scriptstyle [\ell^n]} \\ C & \xrightarrow{f} & A \end{array}$$

For simplicity of notation, we will only consider the first case here. Note that the $C_n \to C$ are geometrically (branched) Galois extensions with an abelian Galois group $G_n \cong (\mathbb{Z}/\ell^n\mathbb{Z})^b$ for $b = 1$ in Case A and moreover, the G_n themselves have an action of σ_q.

Let $M_n = H^1_{\text{ét}}(\overline{C}_n, \mathbb{Z}_\ell)/H^1_{\text{ét}}(\overline{C}, \mathbb{Z}_\ell)$ and define the characteristic polynomials

$$f_n(x) = \det\left(1 - \sigma_q x | M_n\right), g_n = \det\left(1 - \sigma_q x | M_n/M_{n-1}\right).$$

It does not seem to be true that g_n determines g_{n+1} as in Example 1. Nonetheless, the following weaker convergence statement is true. For simplicity again, we assume that q is a prime power such that $q - 1$ is divisible by ℓ but not ℓ^2.

Theorem. *[G.22a, Theorem 12] There is a factorization*

$$f_m(x) = \prod_{n \le m} h_n(x^{\ell^{n-1}})$$

for some polynomials $h_n(x) \in \mathbb{Z}[x]$ independent of m. Moreover, the following congruence

$$h_{n+1}(x) \equiv h_n(x) \pmod{\ell^n}$$

holds so that the limit $h_\infty(x) = \lim_n h_n(x)$ exists inside $\mathbb{Z}_\ell[x]$.

The convergence of the $h_n(x)$ is quite unexpected and striking and I prove it by proving a more general theorem about the convergence of certain sequences of matrices related to non-abelian Iwasawa theory. Note that this more abstract statement can be applied to many more geometric contexts than just our two examples of towers of curves above although I do not pursue this in our paper. It applies to any tower of varieties with an action of an abelian group such that the Frobenius action on the cohomology has a "large" orbit. For instance, one could take hypersurfaces of the form

$$f(x_0^{\ell^n}, \dots, x_n^{\ell^n}) = 0 \subset \mathbb{P}^n_{\mathbb{F}_q}.$$

To keep notation simple, the following is a special (yet non-trivial) case of the general ℓ-adic convergence theorem.

Theorem. *[G.22a, Theorems 7,8] Let $F(t)$ be a $r \times r$ matrix with entries in $\mathbb{Z}_\ell[t]$. Suppose that q is a prime such that $q-1$ is divisible by ℓ but not ℓ^2. For each $n \geq 1$, define the matrix*

$$A_n = \prod_{i=1}^{\ell^{n-1}} F(\zeta_{\ell^n}^{q^i})$$

with characteristic polynomial $p_n(x)$. Then, the limit $p_\infty(x) = \lim_n p_n(x)$ exists and more precisely, the following congruence is true:

$$p_{n+1}(x) \equiv p_n(x) \pmod{\ell^n}.$$

3.2 *The density of polynomials with a given factorization type*

Let us identify the points of $\mathbb{P}^n$ over some field k with univariate polynomials (up-to scaling). Given a "factorization type" τ, we can form the subset of polynomials X_τ in $\mathbb{P}^n(k)$ with the given factorization type τ. For instance, τ might correspond to polynomials that factor completely over k. A natural question is to ask about the relative proportions of X_τ as we vary over τ. This question is highly sensitive to the base field - over an algebraically closed field all polynomials split completely while over a finite field $\mathbb{F}_q$, the limiting proportions (for large q) are given by the Chebotarev theorem.

In on-going work with Yifan Wei and John Yin, we aim to answer this question over k a non-archimedean local field like $\mathbb{Q}_p$. In this setting, we mean "Haar measure" by proportion and the interest is in the exact formulas $\rho(\tau; p)$ for any fixed p. In [BCFG22, Conjecture 1.2], they conjecture that $\rho(\tau; p)$ is a rational function in p that satisfies the remarkable symmetry $\rho(\tau; 1/p) = \rho(\tau; p)$. In our current work, we generalize this conjecture (suitably modified) to any generically Galois, finite map $X \to Y$ over a p-adic field and prove it in many (if not all!) cases.

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